

CSHPM/SCHPM
Annual Meeting/Colloque Annuel
Ryerson University
Toronto, Ontario
Final Program

Sunday (2017-05-28):

9:00-9:15 Welcome by Dirk Schlimm (President of CSHPM/SCHPM)(Room: Kerr South 251)

Session 1a: French Influences on Portuguese Mathematics

Room: Kerr South 251; Presiding: Eisso Atzema

9:15-9:45 Maria Zack (Point Loma Nazarene University), *Manuscript Transmission of Mathematical Knowledge in 18th Century Portugal*

9:45-10:15 Luis Saraiva (CMAFCIO/University of Lisbon), *Maurice Fréchet in Portugal, 1942*

Session 1b: Modern Logic

Room: Kerr South 369; Presiding: Thomas Drucker

9:15-9:45 Eamon Darnell (University of Toronto) & Aaron Thomas-Bolduc (University of Calgary), *Takeuti's Well-Ordering Proof: is it Fine, Finitistically?*

9:45-10:15 Philippos Papayannopoulos (University of Western Ontario), *The Open Texture of 'Real Number Algorithms'*

COFFEE BREAK

**Session 2: New Perspectives on Logic in the Nineteenth Century,
from Kant to Russell (Joint Session with CPA)**

Room: Kerr South 369; Presiding: Greg Lavers

10:30-11:00 Sandra LaPointe (McMaster University), "Logic in the 19th Century, before Frege"

11:00-11:30 Dirk Schlimm (McGill University), "Practices of 19th-century Logic"

11:30-12:00 Erich Reck (University of California at Riverside), "The Logic in Dedekind's Logicism"

LUNCH BREAK

12:00-14:00 Executive Council Meeting (Room: Jorgenson 440)

**Session 3: New Perspectives on Logic in the Nineteenth Century,
from Kant to Russell (Joint Session with CPA)**

Room: Kerr South 369; Presiding: Erich Reck

14:00-14:30 Nicholas Griffin (McMaster University), “Russell and Hilbert and Kant and Geometry”

14:30-15:00 Jamie Tappenden (University of Michigan), “Frege, Carl Snell and Romanticism; Fruitful Concepts and the ‘Organic/Mechanical’ Distinction”

15:00-15:30 Nick Stang (University of Toronto), “Anti-Psychologism in Context”

COFFEE BREAK

Session 4a: Mathematics Education in Russia (Parallel Session)

Room: Kerr South 251; Presiding: Eisso Atzema

15:45-16:15 Mariya Boyko (University of Toronto), “Socialist Competition in the Soviet Mathematics Curriculum Reform of the 1960s and 1970s”

16:15-16:45 Inna Tokar (City College of New York), “History of Schools for Mathematically Talented Students in the Former Soviet Union”

Session 4b: Philosophy of Mathematical Practice (Parallel Session)

Room: Kerr South 369; Presiding: Thomas Drucker

15:45-16:15 Nicolas Fillion (Simon Fraser University), **Constructive and Conceptual Mathematics**

16:15-16:45 Zoe Ashton (Simon Fraser University), **Mathematical Proof and the Contact Between Minds**

17:00-19:00 President’s Reception

Monday (2017-05-29):

Session 5: Special Session—History of 18th-Century Mathematics

Room: Victoria 205; Presiding: Craig Fraser

8:45-9:15 Amy Ackerberg-Hastings (Independent Scholar), *John Playfair’s Approach to “the Practical Parts of the Mathematics”*

9:15-9:45 Marion W. Alexander (Houston Community College), *What Mathematics Rittenhouse Knew*

9:45-10:15 Duncan Melville (Saint Lawrence University), *John Marsh and the curious world of decimal arithmetic*

COFFEE BREAK

Session 6: Special Session—History of 18th-Century Mathematics

Room: Victoria 205; Presiding: Pat Allaire

10:30-11:00 Christopher Baltus (SUNY College at Oswego), *Monge's Descriptive Geometry in Context*

11:00-11:30 Colin McKinney (Wabash College), *The Four Curves of Alexis Clairaut*

11:30-12:00 Robert Bradley (Adelphi University), *Formal Power Series and the Foundational Problem*

LUNCH BREAK

12:00-14:00 Annual General Meeting (Room: Victoria 205)

14:00-15:00 Annual CSHPM Kenneth O. May Lecture: William Dunham (Bryn Mawr College), *A Tale of Two Series* (Room: Victoria 205)

Session 7: Special Session—History of 18th-Century Mathematics

Room: Victoria 205; Presiding: Robert Bradley

15:15-15:45 Daniel J. Curtin (Northern Kentucky University), *E 133: On the Surface Area of Scalene Cones*

15:45-16:15 Lawrence D'Antonio (Ramapo College of New Jersey), *"The geometer spends his whole life with his eyes closed": Diderot contra d'Alembert*

16:15-16:45 William W. Hackborn (University of Alberta—Augustana Campus), *Euler Goes Ballistic under Frederick the Great*

Tuesday (2017-05-30):

Session 8a: Mathematical Physics & Mathematics in Canada (Parallel Session)

Room: Victoria 503; Presiding: Craig Fraser

8:45-9:15 Roger Godard (Department of National Defense), "Boltzmann and Vlasov"

9:15-9:45 Alessandro Selvitella (McMaster University), *On Francis Ronald Britton and his Legacy at McMaster University*

9:45-10:15 David Orenstein (University of Toronto), *The Canadian International Congresses of Mathematicians: Toronto 1924, Vancouver 1974*

Session 8b: History of Logic (Parallel Session)

Room: Victoria 503; Presiding: Dirk Schlimm

8:45-9:15 Greg Lavers (Concordia University), *Did Frege solve one of Zeno's Paradoxes?*

9:15-9:45 Valérie L. Therrien (University of Western Ontario), *The Axiom of Choice as Paradigm Shift: The Case for the Distinction between the Ontological and the Methodological Crisis in the Foundations of Mathematics*

9:45-10:15 Thomas Drucker (University of Wisconsin–Whitewater), *Mathematicians and Their Logics*

COFFEE BREAK

Session 9a: Pre-18th Century Mathematics (Parallel Session)

Room: Victoria 503; Presiding: Amy Ackenberg-Hastings

10:30-11:00 Joel Silverberg (Roger Williams University) & Kim Plofker (Union College), *The Most Obscure and Inconvenient Tables that have ever been Constructed?*

11:00-11:30 Robert Thomas (University of Manitoba), *What is Stated and Almost Always Proved in the Spherics of Theodosios*

11:30-12:00 Andrew Perry (Springfield College), *Recreational and Practical Mathematics of Michael of Rhodes*

Session 9b: Notions of Mathematics (Parallel Session)

Room: Victoria 504; Presiding: Greg Lavers

10:30-11:00 Scott Edgar (Saint Mary's University), *A Defense of Hermann Cohen's Principle of the Infinitesimal Method*

11:00-11:30 Teresa Kouri (Ohio State University), *Ante Rem Structuralism and the No Naming Constraint*

11:30-12:00 Phil Bériault (University of Waterloo), *A Non-Error Theory Approach to Mathematical Fictionalism*

LUNCH BREAK

Session 10a: 19th-Century Mathematics (Parallel Session)

Room: Victoria 503; Presiding: David Orenstein

14:00-14:30 Craig Fraser (University of Toronto), *The Culture of Research Mathematics in 1860s Prussia: Adolph Mayer and the Theory of the Second Variation in the Calculus of Variations*

14:30-15:00 Gabriel Larivière (Simon Fraser University), *On Cauchy's Early Rigourization of Complex Analysis*

15:00-15:30 Eisso J. Atzema (University of Maine), *In Search of Joseph-Émile Barbier (1839-1889): A Bio-bibliographical Sketch*

Session 10b: On Mathematics Education (Parallel Session)

Room: Victoria 504; Presiding: Dirk Schlimm

14:00-14:30 Eugene Boman (Pennsylvania State University—Harrisburg Campus), “Where is the Differential in Differential Calculus?” (Cancelled)

14:30-15:00 Derek Postnikoff (University of Saskatchewan), “Platonism and Plagiarism: Imitation, Collaboration, and Attribution in Mathematics Education” (Cancelled)

15:00-15:30 Parzhad Torfehnezhad (Université de Montréal), *Empirical versus Rational Abstraction: A Reflection on Carnap's notion of abstraction*

15:30-16:00 Bernd Buldt (Indiana University-Purdue University Fort Wayne), “Mastering New Mathematical Concepts”

16:00-16:05 CONCLUDING REMARKS (Room: Victoria 504)

End 2016 CSHPM/SCHPM Annual Meeting

ABSTRACTS

Amy Ackerberg-Hastings, Independent Scholar (aackerbe@verizon.net), **John Playfair’s Approach to “the Practical Parts of the Mathematics”**

Best known for *Elements of Geometry* (1795) and *Illustrations of the Huttonian Theory of the Earth* (1802), University of Edinburgh mathematics and natural philosophy professor John Playfair (1748–1819) also wrote several dozen books, expository articles, and opinion pieces as individual publications or for *Philosophical Transactions of the Royal Society of London*, *Transactions of the Royal Society of Edinburgh*, and *Edinburgh Review*. Most of these works have been digitized and are readily available for study. In contrast, Playfair’s 1793 *Prospectus of a Course of Lectures on Some of the Practical Parts of the Mathematics* has nearly disappeared, with as few as eight surviving copies. In this talk, I will consider what we can learn from this document, which consists of a nineteen-page list of potential topics. Despite its brevity, the *Prospectus* includes a number of themes and priorities that echoed the activities of 18th-century mathematicians. These also recurred throughout Playfair’s writing and teaching, such as in his interest in the figure of the earth problem and his distinctive way of thinking about the roles of pure and applied mathematics.

Marion W. Alexander, Houston Community College (marion.alexander@hccs.edu), **What Mathematics Rittenhouse Knew**

David Rittenhouse (1732–1796), an American astronomer, engineer, craftsman, and public servant in Philadelphia, also dabbled in mathematics. Since his mathematical papers offer no proofs of their claims (most of which had been proved decades before), the question of Rittenhouse’s mathematical ability has remained a mystery. It is often considered in light of how much formal schooling he had (very little) and to what few resources an American of those Revolutionary and Early Federalist times might have access. Rittenhouse was Librarian of the American Philosophical Society, from 1775 until 1791...This talk will explore what a unique vantage point that position gave him. Hints from some scraps of his mathematical work, as well as records of his correspondence with others (chiefly Jefferson), help us understand what Rittenhouse was seeing and what he may have known.

Zoe Ashton, Simon Fraser University (zashton@sfu.ca), **Mathematical Proof and the Contact Between Minds**

While argumentation theory has been adopted in the study of mathematical proofs, much of the research has been focused away from the inclusion of audiences. This lack of focus on audiences stems from a claimed distinction between mathematics and arguments. In this paper, I argue that there is room for audiences in mathematical proof practice. By clarifying Perelman and Olbrechts-Tyteca’s definition of demonstration it becomes clear that what they banned from their theory of argument was formal derivations, not mathematical proofs as they are practiced. By viewing proofs as audience-centered arguments, we can consider the role that the mathematical community plays in problem choice. In addition to the mathematician’s belief in a problem’s intrinsic worth or beauty, we can add the importance of the foundations of argumentation—the contact between minds. The attempt to establish this contact helps to explain certain types of mathematical fads that rely on social aspects rather than intrinsic qualities like problem depth, number of open problems, or beauty.

Eisso J. Atzema, University of Maine (eisso.atzema@maine.edu), **In Search of Joseph-Émile Barbier (1839-1889): A Bio-bibliographical Sketch**

Trained and initially employed as an astronomer, Emile Barbier is probably best known in mathematics for a theorem about curves of constant width now called after him. It is less well known that for most

of his adult life Barbier suffered from severe religious delusions and was institutionalized in some way or other for at least two decades. All the while, he produced mathematical papers of limited scope, but sometimes surprising depth. In this talk, I will sketch the various turns of Barbier's life and analyze the nature of his work.

Christopher Baltus, SUNY College at Oswego (christopher.baltus@oswego.edu), **Monge's Descriptive Geometry in Context**

Gaspard Monge (1746–1818) was at the center of science, mathematics and industry, and in education in those fields, from 1767. He became a founder of the École Polytechnique in 1794. His descriptive geometry, in the curriculum at the Polytechnique, distills essential ideas from his background in military engineering and his pioneering work in the differential geometry of surfaces. A few examples selected from materials prepared, around 1795, for use in the Écoles Normales and the Polytechnique, are intended to illuminate the methods and objectives of Monge's descriptive geometry.

Phil Bériault, University of Waterloo (pberiaul@uwaterloo.ca), **A Non-Error Theory Approach to Mathematical Fictionalism**

Mary Leng has published many spirited, insightful defences of mathematical fictionalism, the view that the claims of mathematics are not literally true. I offer as an alternative an anti-realist approach to mathematics that preserves many of Leng's valuable insights while ridding fictionalism of its most unpalatable feature, the claim that substantive mathematical claims are "in error." In making my argument, I first present the virtues of Leng's fictionalism by considering how she defends it against influential objections due to John Burgess. Leng's view is roughly that indispensability of science is necessary but not sufficient for believing in the reality of something, and that philosophical analysis can make clear why some things, including mathematics, are necessary for science but not real. I suggest we can accept this without adopting a species of error theory. Marrying features of Leng's view with constructivism, a quite different sort of antirealism about mathematics, allows us to: maintain that mathematical assertions are (at least often) literally true but that it is a mistake to understand them as referring to abstract entities; to be antirealists about mathematics; and to make use of the fictionalist toolkit Leng supplies for explaining why mathematics is indispensable even if not real.

Mariya Boyko, University of Toronto (mariya.boyko12@gmail.com), **Socialist Competition in the Soviet Mathematics Curriculum Reform of the 1960s and 1970s**

In 1958 the Soviet government led by Nikita Khrushchev initiated a major reform of education in order to bridge the gap that then existed between the school curriculum and the practical needs of the state. Prominent mathematicians and educators (including Andrei Kolmogorov) were involved in re-writing the mathematics curriculum. However, the content of the new curriculum proved to be unsuitable for the general audience of students who were not highly interested in theoretical mathematics. There were many academic factors that influenced this outcome, but it is also important to recognize the ideological context in which the curriculum reform was taking place. Socialist competition was a prominent ideological phenomenon in the 1950s and 1960s and influenced both the social and academic life of the state. We analyze the role of socialist competition in the mathematics education reforms, a subject that has been neglected in the existing historical literature. We consider socialist competition on international, interstate and interpersonal levels, and examine concrete examples of socialist competition in high school and elementary school settings.

Robert Bradley, Adelphi University (bradley@adelphi.edu), **Formal Power Series and the Foundational Problem**

The differential calculus became widely known to Continental mathematicians through l'Hôpital's *Analyse* in 1696. As the 18th century progressed, the interest in providing a coherent foundation for Leibniz' calculus became more intense. By the close of century, it appeared that such a foundation might arise from the consideration of formal power series. I will examine this evolution, focusing primarily on the work of Euler and Lagrange.

Bernd Buldt, Indiana University—Purdue University Fort Wayne (buldtb@ipfw.edu), **Mastering New Mathematical Concepts**

In 2011, Frank Quinn proposed an analysis of mathematical practice that is informed by both an analysis of contemporary mathematics and its pedagogy. Taking this account as our starting point, we can characterize the current mathematical practice to acquire and work with new concepts as a cognitive adaptation strategy that, first, emerged to meet the challenges posed by the growing abstractness of its objects, and which, second, proceeds according to the following three-pronged approach: (1) sever as many ties to ordinary language as possible and limit ordinary language explanations to an absolute minimum; (2) introduce axiomatic definitions and bundle them up with a sufficient number of examples, lemmata, propositions, etc. into small cognitive packages; (3) practice hard with one new cognitive package at a time. Drawing on research in cognitive science, and especially in mathematics education, I will argue that results from both areas provide supporting evidence for the effectiveness of this mathematical practice.

Daniel J. Curtin, Northern Kentucky University (curtin@nku.edu), **E 133: On the Surface Area of Scalene Cones**

Leonhard Euler in his paper (E133, 1750) *De superficie conorum scalenorum, allorumque corporum conicorum* (On the surface area of a scalene cone and of other conical bodies) gives solutions to the problem of the surface area of a scalene cone, i.e., a cone whose base is a circle, but whose vertex is not over the center of the circle; and then extends the approach to other base curves. The speaker has translated this paper and will discuss Euler's solution. Euler comments that the problem had been addressed previously by Leibniz and Varignon, but their solutions were flawed. Finding these solutions is in progress.

Lawrence D'Antonio, Ramapo College of New Jersey (ldant@ramapo.edu), **“The Geometer Spends his Whole Life with his Eyes Closed”: Diderot contra d'Alembert**

Denis Diderot and Jean le Rond d'Alembert were two towering figures of the French Enlightenment. They worked together for many years as editors of the *Encyclopédie*, until d'Alembert withdrew from the project in 1758. During this period they fought together against the monarchy, the Jesuits, and other forces aligned against the Enlightenment. In this talk we focus on an issue that divided them; the nature of mathematics. Diderot had studied mathematics with great interest in his younger years. But starting with his *Lettre sur les aveugles* of 1749 he criticized mathematics as a closed loop, divorced from the world of sense experience (the title quote comes from this work). On the other hand, for d'Alembert's mathematics always represented a touchstone of rational thought. We examine the philosophical break between the two in light of larger issues, such as d'Alembert's criticism of 'metaphysical reasoning'.

Eamon Darnell, University of Toronto (eamon.darnell@mail.utoronto.ca) & Aaron Thomas-Bolduc, University of Calgary (athomasb@ucalgary.ca), **Takeuti's Well-Ordering Proof: is it Fine, Finitistically?**

In his seminal book *Proof Theory* (originally published, 1975), Takeuti gives a constructive proof of the well-ordering of the ordinal notations in Cantor normal form less than ϵ_0 . These are important for

Gentzen's proof of the consistency of first-order arithmetic. We give a brief description of the proof before discussing first, a couple of small errors, and second the importance this proof to continuing work on finitistic and constructivist mathematics. With respect to the first, we explain how the errors can be corrected without affecting Takeuti's result. With respect to the second, we draw some limits to how strictly finitist the proof is. This is an important issue which, to our knowledge, has not been adequately addressed. Finally, we conclude by discussing some other aspects of our larger project regarding this proof, and why we think our approach and analysis may be important both to proof theory and the philosophy of mathematics.

Thomas Drucker, University of Wisconsin–Whitewater (druckert@uww.edu), **Mathematicians and Their Logics**

There is a good deal of work being done in the twenty-first century about rival logics to what is thought of as the classical version. It is not just a matter of first-order versus second-order, but logics that involve different assumptions and conclusions. The historical record indicates that mathematicians have been perfectly happy with different logics, even without some of the competitors currently being assessed. This talk will look at logics associated with Plato, Descartes, and Leibniz with an eye toward determining how the present Logikstreit may affect the development of mathematics.

William Dunham, Bryn Mawr College (bdunham@brynmawr.edu), **A Tale of Two Series**

As everyone knows, Leonhard Euler (1707–1783) left an indelible mark on 18th century mathematics. His contributions, in terms of their quantity and quality, were breathtaking. To get some sense of Euler's brilliance, we shall consider a pair of infinite series that caught his interest. The first, from a 1737 paper, was the sum of the reciprocals of the primes, and his investigations here can be regarded as the dawn of analytic number theory. The second, from his 1748 classic *Introductio in analysin infinitorum*, was a peculiar variant of the harmonic series that led to a most unexpected sum. In examining these two series, we look over Euler's shoulder and watch a master at work.

Scott Edgar, Saint Mary's University (scott.edgar@smu.ca), **A Defense of Hermann Cohen's Principle of the Infinitesimal Method**

In Bertrand Russell's 1903 *Principles of Mathematics*, he offers an apparently devastating criticism of Hermann Cohen's 1883 *Principle of the Infinitesimal Method and its History*. As Russell sees it, Cohen fails to understand how the modern concept of limits undercuts Cohen's claim that calculus is committed to the existence of infinitesimals. Russell holds Cohen up as an example of how reasoning about mathematics goes badly awry in the hands of innumerate philosophers. This paper argues that Cohen's views are not the consequence of simple mathematical mistakes, as Russell thinks they are. Rather, the paper defends an interpretation of Cohen according to which his idiosyncratic views on infinitesimals are a consequence of his deepest philosophical commitments, commitments which are both rationalist and Leibnizian. The interest of this defense of Cohen is that it reveals a deep philosophical assumption of Russell's objections that Russell does not recognize, and that, to the extent that we find Russell's objections persuasive, we share.

Nicolas Fillion, Simon Fraser University (nfillion@sfu.ca), **Constructive and Conceptual Mathematics**

This paper revisits the tension between constructive and non-constructive approaches to mathematics by emphasizing that the key lesson to be drawn from the two revolutions that have shaped modern mathematics (the 19th-century revolution and the computer revolution) is that good mathematical

practice results from integrating the computational and conceptual styles of mathematics.

Craig Fraser, University of Toronto (craig.fraser@utoronto.ca), **The Culture of Research Mathematics in 1860s Prussia: Adolph Mayer and the Theory of the Second Variation in the Calculus of Variations**

In 1868 Adolph Mayer published an important article in Crelle's journal on the calculus of variations, a branch of analysis that was intensively studied in the nineteenth century. Mayer investigated the conditions that must be satisfied to ensure that a given function that satisfies the Euler differential equation is a genuine maximum or minimum. The impetus for this article originated with research on the second variation published by the Königsberg mathematician Carl Jacobi in the 1830s. Mayer's result was only one among a large range of new discoveries in analysis, not to mention the whole of mathematics. The thesis of this paper is that it nonetheless provides an illuminating case study of how the collective outlook of the mathematical research community had changed in the nineteenth century. One would not have encountered the kind of theoretical concerns that occupied Mayer and his contemporaries in the work of the Enlightenment masters of analysis. At a fairly concrete level, in a central part of mathematics, the intellectual orientation of researchers had moved away significantly from the point of view that had prevailed a century earlier.

Roger Godard, Department of National Defense (rgodard3@cogeco.ca), **Boltzmann and Vlasov**

This work concerns David Hilbert's sixth problem in mathematical physics and the kinetic theory of gases. Influenced by Maxwell, Boltzmann published in 1872, a fundamental equation describing the evolution of the density of probability in six-dimensional space of a particle velocity and position as a function of time. It is a non-linear integro-differential equation, difficult to solve. Boltzmann proved that it is an irreversible process towards equilibrium. We shall analyze Boltzmann's equation and its iterative solution by Chapman and Enskog in 1916-1917, and the hot scientific discussions concerning the reversibility in time of a process (Newton's laws of motion), and the irreversibility of the Boltzmann equation. Finally, we present Anatoly Vlasov, a Russian physicist who adapted the Boltzmann equation to ionized gases in 1938.

Nicholas Griffin, McMaster University (ngriffin@mcmaster.ca), **Russell and Hilbert and Kant and Geometry**

The paper contrasts a surprisingly Kantian strain in Hilbert's work on geometry at the end of the nineteenth century with a surprisingly formalist strain in Russell's at about the same time.

William W. Hackborn, University of Alberta—Augustana Campus (hackborn@ualberta.ca), **Euler Goes Ballistic under Frederick the Great**

In 1745, while serving under Frederick the Great at the Berlin Academy of Sciences, Leonhard Euler translated Benjamin Robins' *New Principles of Gunnery* (1742) into German, adding an extensive mathematical and scientific commentary of his own. Later Euler published "Recherches sur la véritable courbe que décrivent les corps jettés dans l'air ou dans un autre fluide quelconque" (E217) in *Memoirs of the Berlin Academy of Sciences* (1755). In these two works, Euler builds on the investigations of Robins in *New Principles*, Newton's studies of fluid drag in Book 2 of his *Principia*, and a paper by Johann Bernoulli in *Acta Eruditorum* (1719) on the motion of a projectile subject to fluid drag varying as a power of the projectile's speed. This talk will focus on Euler's contributions to ballistics, which resulted in a method (used as recently as World War II) that artillery officers could employ in the field to calculate the significant attributes of a mortar shot.

Teresa Kouri, Ohio State University (kouri.2@buckeyemail.osu.edu), **Ante Rem Structuralism and the No Naming Constraint**

Tim Raz has presented what he takes to be a new objection to Stewart Shapiro's ante rem structuralism (ARS). Raz claims that ARS conflicts with mathematical practice. I will explain why this is similar to an old problem, posed originally by John Burgess in 1999 and Jukka Keränen in 2001, and show that Shapiro can use the solution to the original problem in Raz's case. Additionally, I will suggest that Raz's proposed treatment of the situation does not provide an argument for the in re over the ante rem approach.

Sandra Lapointe, McMaster University (sandra.lapointe@me.com), **Logic in the 19-th Century, before Frege**

The idea that Kant's views on logic played a substantial role in shaping our understanding of the discipline today as well as its place within the theory of knowledge is likely to be, at best, controversial. For one thing, the idea clashes with the standard accounts of the development of logic over the course of the modern era. This paper is mostly devoted to criticizing standards account of the development of logic over the modern period and of Kant's contribution to it. My aim is to explain why the standard accounts can be misleading, give what I take to be sufficient reasons to revise some common beliefs concerning the context in which Kant's views on logic emerged and, after sketching Kant's own views, to provide evidence of their impact on his successors.

Gabriel Larivière, Simon Fraser University (glarivie@sfu.ca), **On Cauchy's Early Rigourization of Complex Analysis**

In this talk, I look at an important development in the history of Western mathematics: Cauchy's early (1814-1825) rigourization of complex analysis. I argue that his work should not be understood as a step in improving the deductive methods of mathematics but as a clear and systematic stance about the subject and language of mathematics. Cauchy's approach is contrasted with Laplace's use of the "notational inductions," especially in the calculation of various integrals of complex functions. I suggest that Laplace's techniques are partly explained by the influence of some of Condillac's ideas about the language of algebra and abstract quantities. Cauchy's opposition is then not to be seen as stemming from a comeback of geometric and synthetic methods, as Argand's theory might be. Instead, from within this algebraic tradition, Cauchy rejected the key Condillacian doctrines that algebra is about abstract quantities and that its grammar provides means of discovering new mathematical truths. He thereby shortened the gap between arithmetic and algebra and fruitfully extended his approach to complex analysis, a significant way in which he contributed to the arithmetization of calculus. I finish by discussing lessons we can draw about how mathematical rigour differs from rigour in other sciences.

Greg Lavers, Concordia University (glavers@gmail.com), **Did Frege solve one of Zeno's Paradoxes?**

It is well known that from the development of the calculus to set theory's clarification of the infinite, modern mathematics (barring certain possible anachronisms) solves many of Zeno's paradoxes. Zeno composed a book of, apparently 40, paradoxes, but only a few survive. One of them (described in Plato's Parmenides) is the paradox of the like and the unlike. This is generally viewed as one of the, if not the, weakest of Zeno's surviving paradoxes. Any attempt to evaluate this paradox involves a considerable degree of reconstruction. According to the various reconstructions that have been made, there seems to be nothing at all paradoxical about the assumption that there are many things, and the attempted reductio falters. Gottlob Frege, in his Foundations of Arithmetic, examines what leading mathematicians of his day say about the notion of a unit. Frege identifies considerations forcing one to treat units as

(paradoxically) both distinct and identical. I wish to suggest that Zeno would have thought of a plurality as composed of units, and would have had the conceptual resources to discover the tensions that Frege identifies. On this reconstruction there is something genuinely paradoxical about the likeness paradox, something that took a good deal of conceptual clarity to overcome.

Colin McKinney, Wabash College (mckinnec@wabash.edu), **The Four Curves of Alexis Clairaut**

In 1726, at the age of twelve, Alexis Clairaut presented four families of curves to the Academy of Sciences. The work was later published in the *Miscellanea Berolinensia* in 1734. The first family of curves is similar to those generated by Descartes' mesolabe compass in *La Géométrie*. In this talk, I will discuss aspects of Clairaut's mathematical education, and how they influenced his work. I will also discuss some of the details of his construction of each of the four families of curves, his classification of algebraic curves in terms of degree and genus, and the applications of his curves to the ancient problem of finding means proportional.

Duncan Melville, Saint Lawrence University (dmelville@stlawu.edu), **John Marsh and the curious world of decimal arithmetic**

In 1742, John Marsh published a justly-neglected work, *Decimal Arithmetic Made Perfect*, in which he laid out a complete system of direct calculation of decimal representations of rational numbers. In this talk I will briefly explain parts of his arithmetic, situate author and audience in social and professional context, and use the work to reflect on the status of various forms of numbers among mathematically literate, but non-elite, practitioners.

David Orenstein, University of Toronto (david.orenstein@utoronto.ca), **The Canadian International Congresses of Mathematicians: Toronto 1924, Vancouver 1974**

There have been two International Congresses of Mathematicians held in Canada: the first at the University of Toronto in 1924, the later at the University of British Columbia in 1974. Toronto occurred during the recovery from the devastation of World War I, but Vancouver was dominated by the controlled confrontation of the Cold War. Canada became the host for 1924 by the happy presence of University of Toronto mathematician John Charles Fields (of Fields Medal fame) on the Council of the American Mathematics Society when the AMS dropped the ball on holding the Congress. In early 1970, the International Mathematical Union approached the Canadian Mathematical Congress (later Society) about hosting one, following the smashing success of the Canadian Centennial World's Fair, Expo '67 in Montreal. Through primary sources such as the minutes of the Organising Committee of the 1924 IMC or personal interviews with participants in 1974, I will explore how the political zeitgeist manifested itself in the preparation, the scientific and social programmes, and the aftermath of these Congresses. Sources also include the massive volumes of the published Proceedings, a substantial Congress scrapbook from 1924, issues of the bulletin of the CMC from the early 1970s and the personal papers of H. S. M. Coxeter who was the President of the 1974 ICM.

Philippos Papayannopoulos, University of Western Ontario (fpapagia@uwo.ca), **The Open Texture of 'Real Number Algorithms'**

Although in the domain of the integers, computability is a well-developed mathematical theory, things are less clear in the domain of real numbers. There is no universally accepted theory of computability over the reals so far. Rather, there are incompatible accounts, in the sense that functions which are taken as uncomputable by the one are proved computable by the other, and the opposite. In this paper, I examine from a conceptual point of view two such accounts: the BSS model (an algebraic approach developed

by Blum et al., 1997) and the recursive analysis approach. Both accounts aim to formalize the notion of an ‘algorithmic computation over the reals.’ I argue that in the process of formalizing an intuitive notion, there are three stages generally and that some element of decision is also involved. Furthermore, the intuitive notion can sometimes be disambiguated in more than one (but equally legitimate) ways; that is, it exhibits open texture. I submit that the notion of a ‘real number algorithm’ is a case in point and that the two conflicting models are the result of following two different routes to “sharpen” the pre-theoretic notion. Accordingly, during this process, different idealizations and restrictions are called for and so different formal models of computability arise.

Andrew Perry, Springfield College (perryand@gmail.com), **Recreational and Practical Mathematics of Michael of Rhodes**

The fifteenth century Venetian mariner Michael of Rhodes left behind a journal with ruminations on a wide range of topics including shipbuilding, navigation, and various areas of mathematics. Some of the questions posed and answered are apparently recreational in nature. We will look briefly at the scope of the mathematical topics found in this text, and then study in more depth the math problems which Michael studied out of an apparent interest in mathematics for its own sake. For example, Michael posed a puzzle in which the goal was to determine, based on clues, how much money each player had had at the start of a dice game.

Erich Reck, University of California at Riverside (erich.reck@ucr.edu), **The Logic in Dedekind’s Logicism**

While Frege and Russell are usually seen as the two paradigmatic representatives of logicism, Richard Dedekind was considered another main logicist in the late 19th and early 20th centuries, e.g., by Ernst Schroeder and Ernst Cassirer. Indeed, Dedekind himself presents his project as one of showing that arithmetic is “a part of logic.” But there is a question about what is meant by “logic” in this connection. As Frege was the first to note critically, Dedekind does not formulate basic logical laws, much less the kind of deductive calculus pioneered by Frege. This makes his position hard to pin down and evaluate. At the same time, his foundational works do contain pointers concerning what he took “logic” to include; and we can look at his intellectual context for further hints, including the influence on him by mathematicians such as Gauss, Dirichlet, and Riemann, and the philosophical views to which he was exposed more broadly, e.g., in a lecture course by Hermann Lotze, “German Philosophy after Kant,” that he attended as a student. Taking account of both leads to a reconstruction of Dedekind’s views that makes the notion of function central, locates his views more precisely with respect to 19th-century debates about Kant, and points ahead to later developments in set theory and category theory.

Luis Saraiva, CMAF/Universidade de Lisboa (lmsaraiva@ciencias.ulisboa.pt), **Maurice Fréchet in Portugal, 1942**

The Portuguese Society of Mathematics was founded in December 1940 by a group of mathematicians who had two main aims. On the one hand, they wished to introduce to Portugal those areas of research that were of interest in other countries at the time. On the other, they wished to publicise these new areas and to revitalise Portuguese Mathematics by capturing the interest of university and pre-university youth. In this talk we will contextualize the coming of age of this generation of mathematicians, known in Portugal as “the 40s generation.” We will analyze several aspects of their work, including the journal *Portugaliae Mathematica*, the *Mathematics Gazette*, the foundation of the Portuguese Society of Mathematics and its first years of activities. Our analysis is only made for the period 1936-1945, a crucial period of change in what concerns mathematical activity as an institutionalised practice. Concerning mathematical developments, this is the most interesting period of this age in Portugal, and it coincides

with the stay in this country of António Monteiro after his return from Paris, where he completed his PhD under the supervision of Maurice Fréchet. Monteiro is the decisive figure of this time and the one behind most of the important mathematical innovations in Portugal in the first half of the 20th century. We will see through the correspondence between Monteiro and Fréchet how Fréchet's stay in Portugal was prepared and we will go in some detail about the activities of Fréchet in Portugal and his contact with Portuguese mathematicians.

Dirk Schlimm, McGill University (dirk.schlimm@mcgill.ca), **Practices of 19th-century Logic**

In this talk I will briefly present and discuss two main strands in logic in the 19th century: The algebraic logic of Boole and his followers and Frege's *Begriffsschrift*. In particular, I will focus on the role that the notations played in consolidating these practices, because during the historical developments, both Boole's and Frege's conceptions of logic changed quite substantially, while their notations stayed the same. Moreover, also in the debate between Boole's follower Schroeder and Frege, not so much the expressive power, but the notation was a major issue of contention.

Alessandro Selvitella, McMaster University (aselvite@math.mcmaster.ca), **On Francis Ronald Britton and his Legacy at McMaster University**

In this paper, we outline the career and contributions of Dr. Francis Ronald Britton, along several decades at McMaster University. We start by concisely reviewing the history of McMaster University, with a particular focus on the departments related to mathematics. Then, we discuss Dr. Britton's role in the history and development of the Department of Mathematics at McMaster and look into a few details of his life. We then present his Ma Thesis and PhD Thesis work. Finally, we conclude with some personal memories of Dr. Britton shared by McMaster's faculty members and his active legacy via the Britton Professorship, Britton Scholarships and Britton Lecture Series.

Joel Silverberg, Roger Williams University (joel.silverberg@alumni.brown.edu) & Kim Plofker, Union College (Kim_Plofker@alumni.brown.edu), **The Most Obscure and Inconvenient Tables that have ever been Constructed?**

With an Oxford degree and close ties to both François Viète and Thomas Harriot, Nathaniel Torporley was perhaps one of the most enigmatic contributors to the mathematics of the sixteenth-century. John Aubrey, in his *Brief Lives*, states that "Mr. Hooke affirms to me that Torporley was amanuensis to Vieta; but from whom he had that information he haz now forgot, but he had good and credible authority for it and bids me tell you that he was certainly so... [and that] Mr. Nicholas Mercator assures me that the earle of Northumberland who was a prisoner in the Tower gave also a pension to one Mr Torporley, Salopiensis, a learned man." Torporley published only one mathematical work: *Dicilides Coelometricae, seu valvae astronomicae universale*, frequently described as barely comprehensible, in which he developed two sets of tables relating to spherical trigonometry and astrology, one of which is very strange indeed. Concerning these tables, named Quadrans and Quincunx, Delambre writes: "Cette dernière est composée de cinq parties différentes. Ces deux tables peuvent passer à bon droit pour les plus obscures et les plus incommodes qui aient jamais été construites." De Morgan abandoned his attempt to untangle the structure of the table Quincunx saying, "Those who like such questions may find out the meaning of the other parts of the table." To our knowledge, no one yet has done so, nor does there exist a translation of *Dicilides* into any other language, nor has anyone commented on or explained the theorems developed by Torporley to produce these tables. In this presentation, we focus on the means by which this table was generated, based upon evidence within Torporley's work itself.

Nick Stang, University of Toronto (nick.stang@utoronto.ca), **Anti-Psychologism in Context**

While it is clear that Frege was an anti-psychologist about logic and (at least by the early 1890s) some kind of ‘Platonist’ about thoughts [Gedanken], scholars dispute what kind of Platonist, exactly, he was. My aim in this paper is to shed light on Frege’s anti-psychologism and his Platonism by comparing him to Hermann Lotze, whose anti-psychologistic conception of logic was massively influential on late 19-th and early 20-th century European thought. The Lotze-Frege relation was the subject of a famous exchange between Michael Dummett and Hans Sluga. I begin by arguing, contra Dummett, that Frege’s “17 Kernsätze zur Logik,” far from being anti-Lotzean, in fact expresses a fundamentally Lotzean conception of logic. In the second part of the paper I address what might seem to be the true fault-line between Lotze and Frege: the ontological status of thoughts [Gedanken]. I examine Lotze’s most famous philosophical doctrine, the distinction between existence and validity [Geltung], and how, according to Lotze, this is a kind of Platonism that does not ‘hypostasize’ logical entities (e.g. thoughts). I then draw a comparison between Lotze’s ‘non-hyposstatic’ Platonism and a certain ‘deflationary’ reading of Frege’s abstract ontology (pioneered by Thomas Ricketts and Erich Reck). I argue that the ‘deflationary’ threads in Frege’s thinking about abstract ontology are, plausibly, inspired by Frege’s engagement with Lotze. My thesis is appropriately qualified: to the extent that the ‘deflationary’ reading is correct, Frege is a Platonist of the Lotzean stripe.

James Tappenden, University of Michigan (tappen@umich.edu), **Frege, Carl Snell, and Romanticism; Fruitful Concepts and the ‘Organic/Mechanical’ Distinction**

A surprisingly neglected figure in Frege scholarship is the man Frege describes (with praise that is very rare for Frege) as his “revered teacher,” the Jena physics and mathematics professor Carl Snell. There is more of interest to say about Snell than can fit into one paper, so I’ll restrict attention here to just this aspect of his thought: the role of the concept of “organic,” and a contrast with “mechanical.” Snell turns out to have been a philosophical Romantic, influenced by Schelling and Goethe, and Kant’s *Critique of Judgement*. The paper also goes beyond Snell to explore other figures at Jena, particularly in the salon Snell sponsored and that Frege attended. Here too the “organic/mechanical” contrast, understood in a distinctively Romantic fashion, had reached the status of “accepted, recognized cliché.” More generally, Frege’s environment was more saturated with what we now call “Continental philosophy” than we might expect. (Recently this “Continental” dimension of Frege’s environment has been explored by Gottfried Gabriel and others, with an emphasis on neo-Kantianism and Herbart. This paper develops a different dimension: the speculative idealism informing German Romantic biology.)

Valérie L. Therrien, University of Western Ontario (vtherri@uwo.ca), **The Axiom of Choice as Paradigm Shift: The Case for the Distinction between the Ontological and the Methodological Crisis in the Foundations of Mathematics**

Seldom has a mathematical axiom engendered the kind of criticism and controversy as did Zermelo’s 1904 Axiom of Choice (henceforth, AC). We intend to place the development of the Axiom of Choice in its proper historical context relative to the period often called “the crisis in the foundations of mathematics.” To this end, we propose that the nature of the controversy surrounding AC warrants a division of the *Grundlagenkrise der Mathematik* into two separate horns : a) an ontological crisis related to the nature and status of mathematics itself (*viz.*, the nature of its foundation and the logical paradoxes that surrounded early attempts to logically formalize mathematics); and b) a methodological crisis concerned rather with the nature of mathematical practice (*viz.*, the nature of mathematical proofs). These two strands are inexorably intertwined and, though it is not new to suggest that the controversy surrounding AC was related either to the foundational crisis or to a polemic about the nature of mathematical demonstration, it is perhaps new to state that the question of the validity of AC not only was a central question of this period, but, furthermore, was one of its primary drivers—one which led to a profound paradigm shift in the way we construe mathematical reasoning, whether it has led us down a path of embracing realism/Platonism or intuitionism/pragmatism/constructivism.

Robert Thomas, University of Manitoba (robert.thomas@umanitoba.ca), **What is Stated and Almost Always Proved in the Spherics of Theodosios**

When Len Berggren and I were near publishing our book on Euclid's Phenomena, we published what the Phenomena says in a journal article accessible to those that don't need or want the whole treatise. Its most important part was a list of the enunciations of the theorems as the obvious way to express the contents. I can now venture a summary of the Spherics of Theodosios 'accessible to those that don't need or want the whole treatise.' I intend that summary to be an appendix to the text of this paper. What I want to say here is why, with examples, the summary cannot take the form the Phenomena summary took, just 'a list of the enunciations of the theorems.' Under the influence of the style in which Euclid cast his theorems, all of the propositions of the Spherics are written out in prose generalities using pronouns to avoid repetition of sometimes needed noun phrases. This makes some of them quite incomprehensible. Sometimes the prose enunciation does not include what is stated later and sometimes proved. Sometimes even a setting-out with letters is terribly difficult to understand without a diagram, and the medieval diagrams are no use to me.

Inna Tokar, City College of New York (innatokar@gmail.com), **History of Schools for Mathematically Talented Students in the Former Soviet Union**

This presentation will examine programs for mathematically talented students in the former Soviet Union. Special emphases will be given to the boarding schools for gifted students at Moscow, Novosibirsk, St. Petersburg and Kiev Universities and to established FMSHs (Physics-Mathematics day schools for gifted). The origins and history of education for gifted students in the former Soviet Union will be discussed. Specifically, the following questions will be considered: i) What opportunities were available before special schools were opened? ii) Why and how were these schools organized? iii) What is the nature of and variations among special school curricula and student, faculty, and alumni bodies? To answer these questions, original literature from Russia and Ukraine was reviewed, including scientific publications, educational journals, government and university documents. Interviews were conducted with Soviet-born mathematicians and educators who created and taught at these schools. Results show that prior to these programs, talented students were limited to Mathematics Olympiads and Mathematics Clubs. A catalyst to opening special schools was Khrushchev's Polytechnic reform. These schools were organized by prominent Soviet scientists and mathematicians to provide highest quality of education to gifted students.

Parzhad Torfehnezhad, Université de Montréal (parzhad.torfeh-nezhad@umontreal.ca), **Empirical versus Rational Abstraction: A Reflection on Carnap's notion of abstraction**

In this paper presentation, I argue that there are basically two different philosophical senses of abstraction: a rational and an empirical one. On the one hand, abstraction might be construed as a mind/experience-independent universal process (ideally, a mathematical operation). This process is capable of producing knowledge regardless of the subject matter, be it science, mathematics or even general propositions. I call this view rational abstraction. It has been advocated by Frege and by contemporary neo-Fregeans like Wright, Fine, Cook, and others. On the other hand, there is another conception of abstraction that identifies it with a subjective process based on the perceptual data (and the properties of our cognitive faculty). I call this view empirical abstraction. In this subjective process, perception constitutes the first level of abstraction and the entire subsequent abstractive process proceeds internally. Although empirical abstraction has its roots in Aristotle's philosophy, I argued (in a previously published paper), that this view is rightfully attributed to the German philosopher Rudolf Carnap. Carnap considered abstraction to be the main constructive process when establishing linguistic frameworks. Although many scholars have shed important light on Carnap's work, the significance of his work on abstraction has not been given proper attention.

Maria Zack, Point Loma Nazarene University (mzack@pointloma.edu), **Manuscript Transmission of Mathematical Knowledge in 18th Century Portugal**

In the 1750s, the mathematics taught at the universities in Portugal was much less advanced than what was being taught in neighboring countries. However, there were traces of sophisticated ideas from the mathematics of materials evidenced in Portuguese buildings, particularly those erected after the 1755 earthquake that leveled Lisbon. This talk discusses the role that manuscript material used in military engineering schools may have played in transmitting mathematical information from francophone Europe to Portugal. The author has recently discovered a 1742 manuscript sitting in a little-known library in Paris that may provide one of the “missing links” in explaining how the mathematics of materials became known in Portugal well before the 1772 reformation of the mathematics curriculum taught in Portuguese universities.

Cancelled Talks

Eugene Boman, Pennsylvania State University - Harrisburg Campus (ecb5@psu.edu), **Where is the Differential in Differential Calculus?**¹

A casual glance through the table of contents of a representative sample of the calculus textbooks published since the middle of the twentieth century will quickly convince you that, by and large, these books are all essentially homeomorphic, if not isomorphic to each other. Even the texts generated by the calculus reform movement of the late nineties barely depart from the standard script. The topics are presented in the same order and in much the same fashion. At one level this makes sense. In the standard syllabus the topics build on one another in a logical progression. However a side effect of this is that the historical development of the topics is entirely missing. The fact is that the standard syllabus for the first semester of calculus is so far removed from its 17th century roots that it would be nearly unrecognizable to either Leibniz or Newton. In 2015 the authors began working on a calculus text which will explicitly use history as its organizing theme. Beginning with a purely intuitive understanding of the derivative as a ratio of differentials we will build the computational tools of calculus. After the computational techniques are mastered we will return to foundational and theoretical questions such as the definition of a derivative via the limit. This talk will essentially be a short description of our progress so far.

Derek Postnikoff, University of Saskatchewan (derek.postnikoff@usask.ca), **Platonism and Plagiarism: Imitation, Collaboration, and Attribution in Mathematics Education**

The clear attribution of contributors and sources vitally situates scholarly work within the evolving dialectical system that constitutes a discipline. Plagiarism—broadly understood as a failure of adequate attribution ? is among the most serious charges that can be leveled against a scholar, whether they be researcher, teacher, or student. However, plagiarism is a more difficult concept than is widely understood, as what qualifies as adequate attribution can vary dramatically relative to context. In this paper, I explore issues of attribution and plagiarism within the context of mathematics education. The highly symbolic, systematic, and objective nature of mathematics leads to different attribution requirements than exist in other academic disciplines. I present a series of example scenarios that aim to challenge the understanding of plagiarism in mathematics. These examples tend to focus around the idea that unattributed imitation and collaboration are an important part of student mathematical practices. One’s reaction to these examples is likely to depend on their foundational stance and their corresponding beliefs about the aims of student mathematical practices. I conclude that clearer plagiarism guidelines for math

¹This talk is a report on joint work with Robert Rogers, State University of New York at Fredonia (robert.rogers@fredonia.edu)

students are desirable, but caution that imposing a highly restrictive policy is likely to be damaging to the entire mathematical enterprise.